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Rheology of a confined granular material

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We study the rheology of a granular material slowly driven in a confined geometry. The motion is characterized by a steady sliding with a resistance force increasing with the driving velocity and the surrounding relative humidity. For lower driving velocities a transition to stick-slip motion occurs, exhibiting a blocking enhancement with decreasing velocity. We propose a model to explain this behavior pointing out the leading role of friction properties between the grains and the container's boundary.

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The rheology of granular materials lacks so far of an established unified picture. Granular assemblies show a great diversity of behavior depending on whether they are vibrated, avalanching on a surface, flowing in a hopper or falling in a chute [1]. For example, when submitted to shearing, strain localization is currently observed (shear bands) [2,3] with a rheology exhibiting nonlocal properties [4] or aging phenomena [5,6]. Several authors argue that this complex phenomenology could be due to the presence, at a mesoscopic level, of a disordered contact force network with unraveled dynamical properties [2,7]. Such structures were found to auto-organize in order to resist to external constraint [7]; they could also be at the origin of dramatic effects observed on the mechanical strength that are induced by slight changes in compacity [2,8,9]. On the other hand, one should recognize another source of difficulty lying in the contribution of contact forces modified by plastic deformation or by the influence of the surrounding humidity (capillary bridges). Several conjectures were proposed to rationalize such effects but so far only few experimental studies are available to clarify the situation [5,6,10–12].

Dynamical behavior of slowly driven granular materials was investigated recently by different groups both in compression and/or in shear experiments [3,5,8,13,14]. Here, we investigate the rheology of a granular assembly confined in a cylindrical column and pushed vertically from the bottom. The resistance to vertical motion, as well as the blocking/unblocking transitions, reveal a phenomenology possibly shared by many confined situations, as for example, gouge sheared between two faults [15], pipe flows, compaction under stress or dense granular paste extrusion. A previous investigation of a similar display but in two dimension (2D) [14] has already shown a rich phenomenology. For the present 3D granular assemblies, we observed similar behavior but in this paper, we only report experiments for the most simple dynamical situation, i.e., low friction grains confined in a column with frictional walls. This situation offers a weak coupling with dilatancy effects due to shearing at the walls; therefore, relations between the granular nature of the bulk (i.e., the stress redirection in response to deformation) and the solid friction properties at the walls are clearly revealed. A central issue of this paper is to show to which extent the aging dynamics of contact forces can play a role in the rheology of a slowly driven granular assembly.

The grains are dry, noncohesive, and monodisperse steel beads of diameter $d=1.58$ mm piled into a vertical duralumin cylinder of diameter $D=36$ mm. The column is closed at the bottom by a movable piston avoiding contact with the column (diameter mismatch is 0.5 mm). A force probe of stiffness $k=40\,000$ N m^{-1} is located under the piston and is pushed at a constant driving velocity V (between 5 nm s^{-1} and 100 μm s^{-1}) via a stepping motor [see inset of Fig. 1(a)]. The resistance force F encountered by the piston is

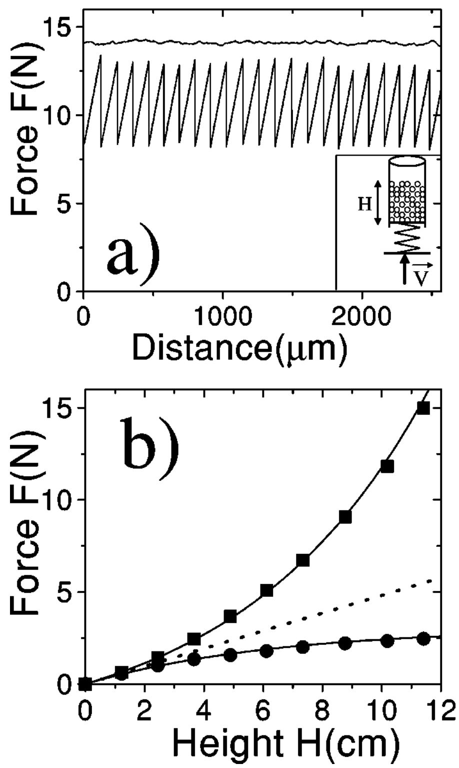

FIG. 1. (a) Resistance force vs the displacement of the stepping motor for $H=4.3R$ steel beads in a duralumin cylinder, for relative humidity $\chi=45\%$ and for $V=30$ nm s^{-1} (stick-slip regime), and $V=100$ μm s^{-1} (steady-sliding regime) shifted by $+5$ N; the inset is a sketch of the experimental display. (b) Mean force in the steady-sliding regime for $V_{up}=16$ μm s^{-1} (squares) and for $V_{down}=16$ μm s^{-1} (triangles) as a function of the height of the packing; the lines are the fits according to Eq. (1); the dotted line is the hydrostatic curve.

measured as a function of time. We monitor also the relative humidity (χ) and the surrounding temperature. We work in the range $35\% < \chi < 75\%$, and also in dry air ($\chi < 3\%$). Actually, except for the dry situation, we do not regulate this last parameter (χ) but we record its values close to the experimental setup. Temperature is kept at (20 ± 1) °C. Note that we actually found no correlation between the force fluctuations and the temperature variations in this range.

We observe two distinct dynamical regimes [Fig. 1(a)]: for high driving velocities, the motion is characterized by a steady sliding and a constant pushing force; for low velocities, the system undergoes a dynamic instability and a stick-slip motion occurs. The transition between these behaviors is similar to the inertial regime of Heslot *et al.* [16], and details will be reported elsewhere. For a vertically pushed granular assembly, the driving force exerted by the piston is screened due to friction with the walls. To evaluate this effect, the mean resistance force is measured as a function of the packing height [see Fig. 1(b)]. For this data set, the driving velocity V corresponds to a steady and continuous sliding of the grains. The resistance force F increases very rapidly with the packing's height H. This strong resistance to motion is due to the horizontal redirection of stresses in association with solid friction at the side walls. Following the standard Janssen's screening picture [9,17], the force F exerted by the grains on the piston can be modeled *in first approximation* by the relation

$$F_\epsilon = \varrho g \lambda \pi R^2 \epsilon \left[\exp\left(\epsilon \frac{H}{\lambda} \right) - 1 \right], \qquad (1)$$

where ϱ is the mass density of the granular material, R is the cylinder radius, and g the acceleration of gravity. The length $\lambda = R/2K\mu$ is the effective screening length, where K is the Janssen's parameter rendering the average horizontal redirection of vertical stresses and μ can either be the dynamic or the static coefficient of friction of beads at the cylinder's wall. When $\epsilon = +1$, friction is fully mobilized downwards (our pushing experiment) and when $\epsilon = -1$, friction is fully mobilized upwards (as in [9]). It is easily seen from Eq. (1) that when $\epsilon = +1$, any slight change in μ or K is exponentially amplified with a drastic influence on the pushing force F. In the case of steel beads, we find that starting from a dense or a loose packing, the final average steady state packing fraction $\overline{\nu}$ does not change; we have $\overline{\nu} \approx 62.5\%$ for all velocities and all relative humidities χ tested. In the steady state regime, the experimental data obtained for a given pushing velocity V can be fitted with relation (1) by adjusting only one parameter, i.e., $p_{+1} = K\mu_d$, where μ_d is the dynamic coefficient of friction at velocity V. For the relative humidity $\chi = 45\%$, we obtain $p_{+1} = 0.140 \pm 0.001$ at $V_{up} = 16\ \mu\mathrm{m\,s^{-1}}$ and $p_{+1} = 0.146 \pm 0.001$ at $V_{up} = 100\ \mu\mathrm{m\,s^{-1}}$.

As a check of consistency, we perform the following dynamical experiment. First, the granular column is pushed upwards in order to mobilize the friction forces downwards and to reach the steady state compacity. Starting from this situation, the friction forces are reversed at the walls by moving the piston downwards at a constant velocity $V_{down} = 16\ \mu\mathrm{m\,s^{-1}}$. Following relation (1), this procedure should imply a change of ϵ from 1 to -1, and consequently, the dynamical force on the piston should decrease from F_{+1} to F_{-1}. In Fig. 1(b) the pushing force F_{-1} is measured for different packing heights H. Inserting the preceding value of $p_{+1}(16\ \mu\mathrm{m\,s^{-1}})$ into (1) with $\epsilon = -1$, we check in Fig. 1(b) that the theoretical expectation agrees quite well with the experimental data of F_{-1} vs H. Note that in a previous study [9], it was found that the Janssen's picture has a general tendency to underestimate slightly the stress below a granular column for an homogeneous packing of glass beads. In the present situation, with low friction steel beads, this model though elementary, seems a fair base for analysis. In fact, seeking for more refined analysis would be useless in the scope of this paper. A central question is still that the fitting parameters $p = K\mu$ extracted from the model do not allow us to distinguish between wall-bead interactions (μ) and bulk properties (K). Actually, from a series of static Janssen experiments performed as in [9], we extract $K\mu_s$. Moreover, we measure independently the static coefficient of friction μ_s of our steel beads on duralumin (in the short time limit) using the sliding angle of a three-bead tripod. We, hence, obtain $\mu_s = 0.170 \pm 0.005$ and $K = 1.08 \pm 0.05$ for $\chi = 45\%$. This K value is consistent with previous measurements done on a granular column at this compacity [9].

For a given height H of beads ($M = 380$ g, i.e., $H = 4.3R$), we study extensively how the pushing force depends on the driving velocity V and on the surrounding relative humidity (χ). All data are shown for a given series for similar humidity values (within 3%). As already mentioned, the motion is characterized by a steady sliding above a critical velocity [Fig. 1(a)]. The mean force level in this regime increases slowly with velocity but surprisingly strongly with χ [Fig. 2(a)]. For example at $V = 100\ \mu\mathrm{m\,s^{-1}}$, the resistance force is raised by 35% for a change of χ from 53% to 72%.

To compare bulk mechanical properties (K) to dynamical frictional properties at the walls (μ_d), we built a special device [the slider, see inset of Fig. 3(a)] designed to apply a constant normal load ($F_N = 2$ N) on three steel beads sliding vertically on the cylinder's wall. Then, the dynamical evolution of the resistance force encountered by the piston pushing a granular material is compared with the slider's friction resistance driven in the same conditions. At a given χ, we also observe clear velocity strengthening for the sliding of individual steel beads, corroborating qualitatively the general trend observed on the granular column. But now, we go one step further by testing directly the possibility of a quantitative agreement in the framework of the Janssen's model. If we compare these data to the values of μ_d extracted from Eq. (1) [see Fig. 3(a)], and still assuming a static value of $K = 1.08 \pm 0.05$ for all velocities, we observe that the increase of μ with V is significantly less important in the case of the granular column than in the case of the slider device.

This means that the increase of F with V cannot be entirely attributed to friction effects at the walls, and that the dynamics may also have an effect on stress transmission in the bulk (i.e., on K). In the framework of a Janssen's analysis this would mean that $K(V)$ would slightly decrease when

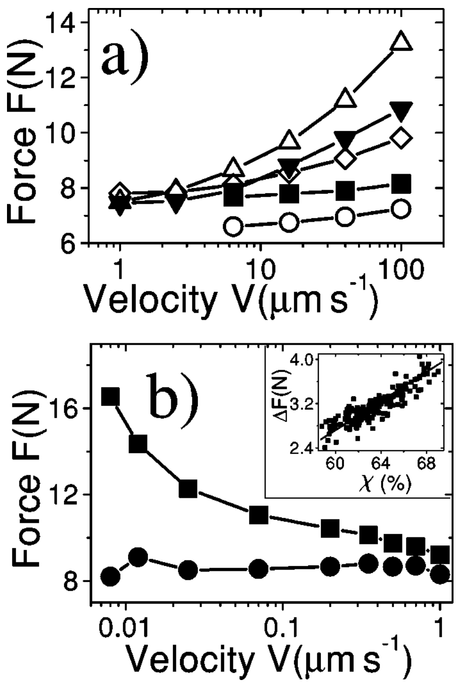

FIG. 2. (a) Mean force in the *steady-sliding regime* as a function of velocity for $H=4.3R$ of steel beads in a duralumin cylinder and for several relative humidities χ [$<3\%$ (circles), 40% (squares), 53% (diamonds), 66% (down triangles), and 72% (up triangles)]. (b) $F_{\min}$ (circles) and $F_{\max}$ (squares) in the *stick-slip regime* as a function of velocity for $H=4.3R$ of steel beads in a duralumin cylinder and for $\chi=45\%$; the inset shows the variation of $\Delta F=F_{\max}-F_{\min}$ with χ for $V=50\ \mathrm{nm\,s^{-1}}$.

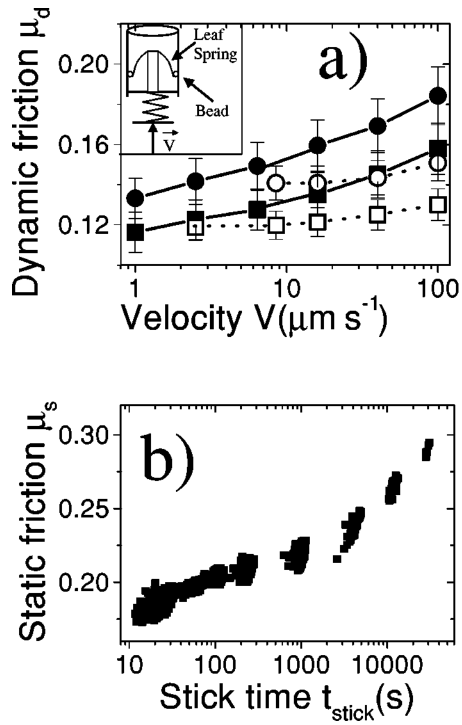

FIG. 3. (a) Dynamic coefficient of friction as a function of velocity for the slider (filled symbols) and for the granular column (empty symbols), for relative humidities $\chi=40\%$ (circles) and $\chi<3\%$ (squares); the inset shows the slider, a constant normal load is applied on the beads by the way of leaf springs. (b) Static friction coefficient as a function of stick time for $H=4.3R$ of steel beads in a duralumin cylinder and $\chi=45\%$.

velocity increases. Note that using a simple Hertz law to estimate contact interactions, we find the penetration depths of steel beads in duralumin to be around $\delta\approx 30$ nm, whereas in the slider case, we estimate $\delta\approx 1\ \mu$m that is the order of duralumin roughness. Due to this difference in deformation, it is possible that the slider and bulk cases are not both in the same loading regime and so that friction laws could be slightly different. Importantly, we have also found that an increase of humidity has quite a strong influence on the friction properties [Fig. 2(a)]. Using the inverted Janssen's model [Eq. (1)] and assuming that the redirection parameter K is unchanged by humidity, we find that the dependence on relative humidity χ in the slider experiment is consistent with the enhancement of the friction forces measured in the granular column. In a future series of experiments, we will try to extend the range of controlled values of χ close to 100%.

Now let us consider slow driving velocities, where the system undergoes a dynamical instability. A stick-slip motion occurs [see, Fig. 1(a)] with a narrow Gaussian distribution of slip force amplitudes (ΔF). In Fig. 2(b), we display the mean maximum and mean minimum resistance forces (resp. $F_{\max}$ and $F_{\min}$) as a function of the driving velocity, for $m=380$ g of beads (height $H=4.3R$), and relative humidity $\chi=45\pm 3\%$. The mean amplitude of the slip events $\Delta F=F_{\max}-F_{\min}$ increases strongly when the velocity reaches values as small as $5\ \mathrm{nm\,s^{-1}}$.

We suggest that this enhanced blocking effect can be simply interpreted as an aging effect of the contacts at the side walls. Friction coefficients of solid on solid contacts are known to evolve logarithmically with waiting time t [18]: $\mu_s(t)=\mu_s^0+\beta_s\log_{10}(t)$. According to Fig. 2(a), we observe no noticeable variation of $F_{\min}$ with velocity, for given height and χ. Therefore, in the following, we consider $F_{\min}$ to be a constant. Starting at the onset of blocking $t=0$, the force exerted by the force probe during a stick event is $F(t)=F_{\min}+kVt$. The time elapsed during a sticking event is

$$t_{stick}=\frac{F_{\max}-F_{\min}}{kV}. \tag{2}$$

The slip occurs when $F(t)$ reaches the maximum force sustainable by the granular material at time t, given by Eq. (1) with $\epsilon=+1$. The aging properties of the friction at the wall are included in the time evolution of the static coefficient of friction $\mu_s(t)$. Then we write $F(t_{stick})=F_{\max}$, i.e.,

$$F_{\max}=\frac{\varrho g\pi R^3}{2K[\mu_s^0+\beta_s\log_{10}(t_{stick})]}\times\left(\exp\left\{2K[\mu_s^0+\beta_s\log_{10}(t_{stick})]\frac{H}{R}\right\}-1\right). \tag{3}$$

This exponential amplification of the logarithmic aging, due to stress redirection at the walls, gives an effective power law : $F_{\max}\sim t_{stick}^{\alpha}$, with $\alpha=2\log_{10}\mathrm{e}\,HK\beta_s/R$. On Fig. 3(b), for $\chi=45\pm3\%$, we display μ_s extracted from Eq. (3) as a function of the time of stick. We assume again $K=1.08$ independent both of the waiting time and the driving velocity. We actually observe a logarithmic aging for waiting times $\sim$3000 s, with a coefficient $\beta_s=1.8\times10^{-2}\pm2\times10^{-3}$, of a magnitude consistent with many previous reports [18]; in the last decade, aging is strongly increased and we have $\beta_s\approx6\times10^{-2}$. Interestingly, such an effect was also found in other granular shearing experiments performed in a quite different geometry [5]. It is not yet clear for us whether this increased aging could be attributed solely to a grain/boundary friction dynamics or is a signature of a long time bulk structuration. Furthermore, consistently with the finding of Refs. [6] and [10], we clearly observe that the aging properties are strongly affected by a variation of the relative humidity χ [see inset of Fig. 2(b)].

To conclude, we investigated the dynamical behavior of a granular column pushed vertically from the bottom. This model experiment is well suited to understand the rheology of slowly driven granular assemblies in confined geometries. At such slow driving velocities, we observe blocking enhancement, aging, and dynamical hardening effects. Our analysis shows that, the nontrivial dynamical properties exhibited by this rheology can be dominantly attributed to the dynamical properties of solid on solid friction at the grains/boundary interface (including a strong dependence on relative humidity). Nevertheless, a quantitative analysis in the framework of the Janssen's model also indicates that a contribution of dynamical structuring effects induced in the bulk cannot be completely excluded.

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